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Mathematics 5

Chance and Uncertainty

Learning Technologies Branch

Alberta LEARNING

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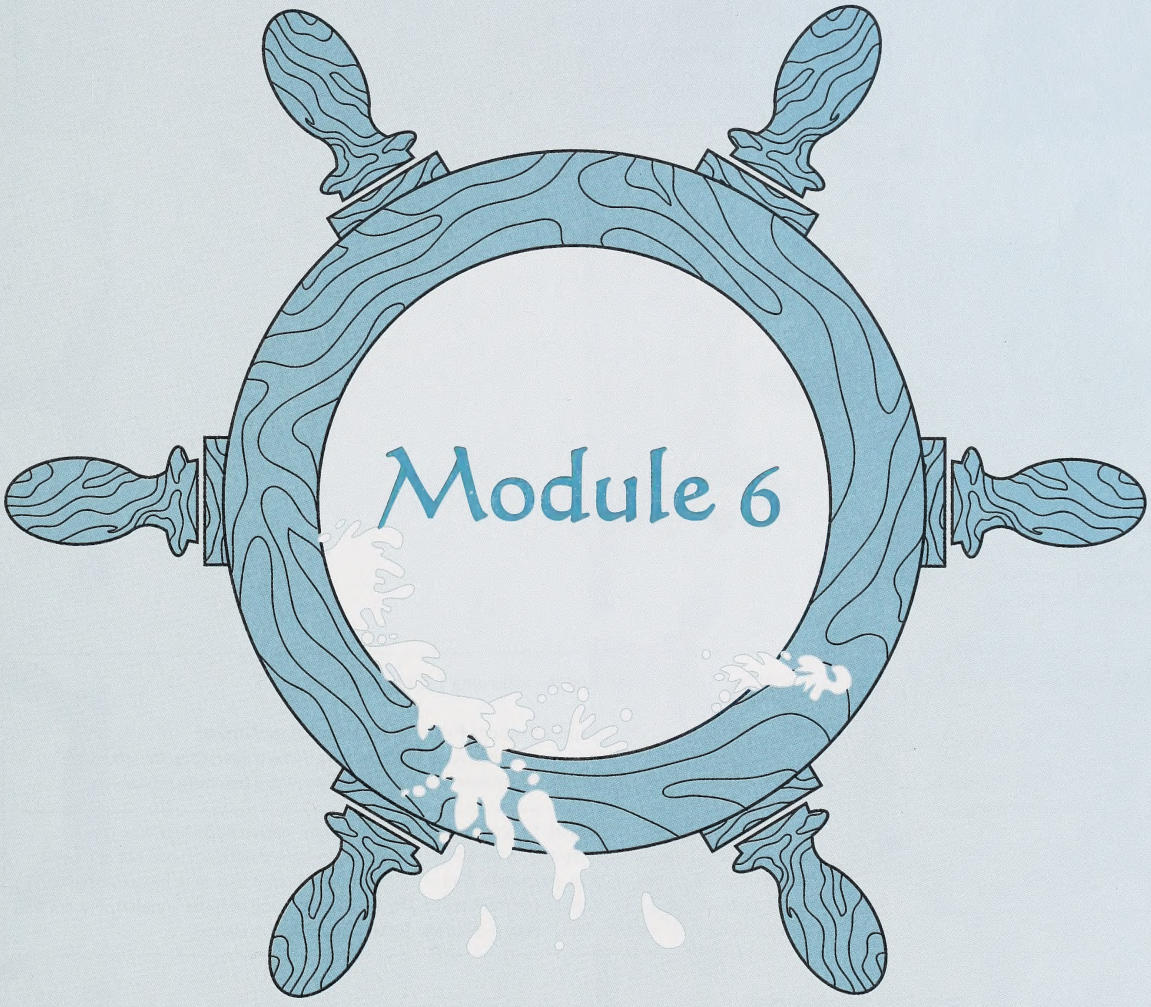
Mathematics 5



Chance and Uncertainty

Learning Technologies Branch

Alberta LEARNING

Mathematics 5
Module 6: Chance and Uncertainty
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

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- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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Welcome Aboard
Mathematics 5

Ahoy! Mathematics 5 contains nine modules.
You should work through the modules in order (from 1 to 9) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Module 1
Whole Numbers

Module 2
Patterns

Module 3
Fractions and Decimals

Module 4
Transformations

Module 5
Data Analysis

Module 6
Chance and Uncertainty

Module 7
Length and Area

Module 8
Volume, Capacity, Mass, and Time

Module 9
2-D Shapes and 3-D Objects

Adventures on the High Seas

Kassidy: Connor, guess what? I noticed an advertisement in the newspaper. A second-hand bookshop just opened, and it's supposed to have some really old books from all over the world.

Connor: Some may even be older than our great-grandparents! Let's go take a look.

Excited about what they might find to read, Connor and Kassidy hurry off to the bookshop.

Connor: Wow! Look at this, Kassidy. Here's an actual journal kept by Captain Quinn almost 300 years ago. It's kind of musty, and parts are hard to read, but look at how much math the captain used!

Kassidy: There are also maps, descriptions of ships, and reports about sea conditions.

Connor: Let's buy it. Figuring out what the journal says will be like unravelling a mystery. Maybe it will have clues about real buried treasure.

From time to time throughout the course, you will work with many of the facts and figures about the days of sailing boats and sea captains that Kassidy and Connor discover.

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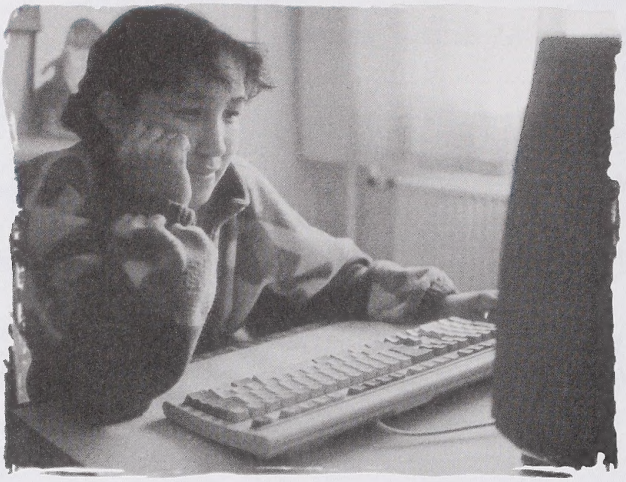
Course Features



BRYAN AND CHERRY ALEXANDER PHOTOGRAPHY

Take the time to look through the Student Module Booklet and the Assignment Booklets and notice the following design features.

- Each module has a Module Overview, Module Summary, and Review.
- Each module has several lessons. Each lesson focuses on a big idea that is central to the topic being learned in the module.
- Each lesson has several activities. An activity in each lesson is related to the high seas adventure theme. The last activity in each lesson is a Challenge Activity.
- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.
- Each module has special exercises that focus on certain mathematical skills. For example, The Numbers in the News project involves a scavenger hunt for samples of math in everyday life. The Keystrokes exercise introduces some "funky features" of the calculator that can be used to explore and practise important number ideas.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

In order to complete the course, you will need a copy of the Mathematics 5 textbook, *Quest 2000: Exploring Mathematics*, the soft-cover book *Quest 2000: Exploring Mathematics: Practice and Homework Book*, a basic four-operation calculator (such as the TI-108 calculator), and various manipulatives (base ten blocks and pattern blocks).

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.

Use your calculator.

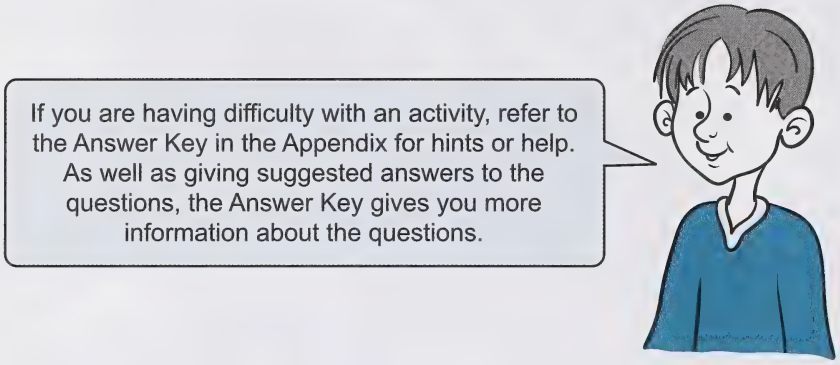
Use the Internet.

Refer to the textbook or the Practice and Homework Book.

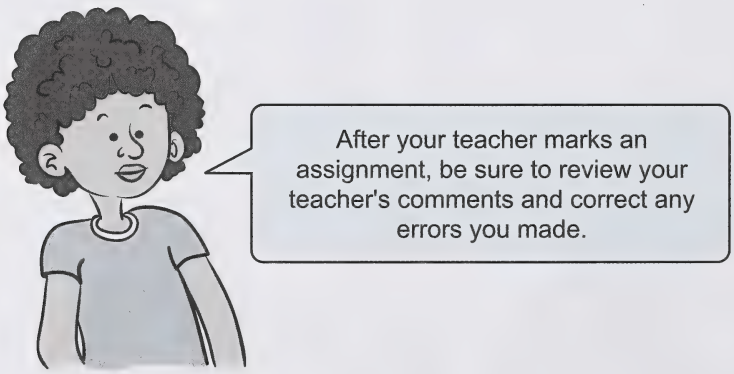
Your guides for this course are Kassidy and Connor.

Assessment and Feedback

The Mathematics 5 course is carefully designed to give you many opportunities to discover how well you are doing. In every activity you will be asked to turn to the Appendix to check your answers. Completing the activities and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.


If you are having difficulty with an activity, refer to the Answer Key in the Appendix for hints or help. As well as giving suggested answers to the questions, the Answer Key gives you more information about the questions.

Twice in each module you will be asked to give your teacher your completed assigments to mark. Your teacher will give you feedback on how you are doing.



There will be a final test at the end of the course. You can prepare for the final test by completing the Review at the end of each module.

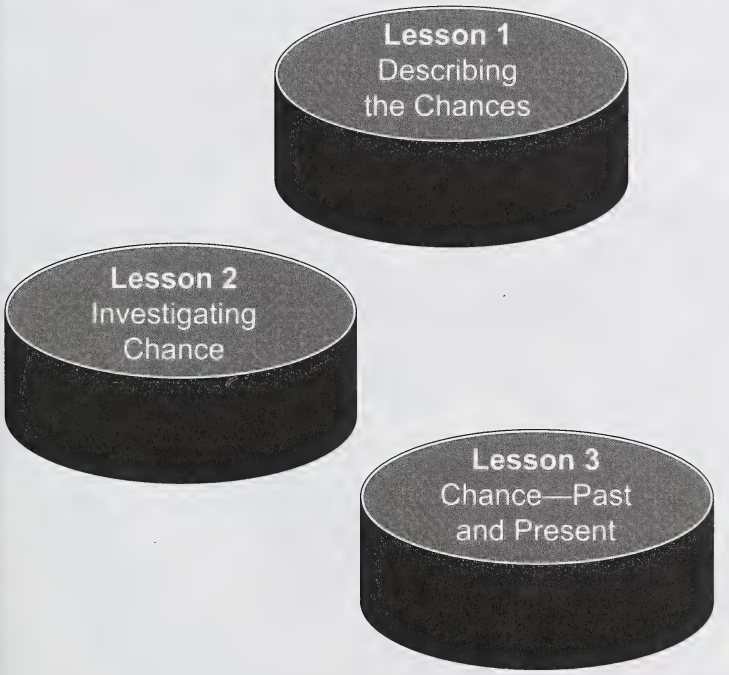
Module Overview

Are you a hockey fan? What is your favourite NHL team? What do you think their chances are of winning the Stanley Cup next season? Is it likely or unlikely? What is the **probability**?

What is the weather forecast for this weekend? Will it rain or snow? What is the probability of precipitation?

The previous examples show that probability is an important part of daily life.

In this module you will explore the vocabulary and mathematics of probability. You will describe the outcomes of everyday events and how likely they are to occur. You will play games and conduct experiments using spinners, number cubes, and other objects to demonstrate and develop the principles of probability.



Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 6A

Lesson 1 Assignment	30 marks
Lesson 2 Assignment	30 marks

Assignment Booklet 6B

Lesson 3 Assignment	30 marks
Numbers in the News	10 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 6A to your teacher before you begin Lesson 3. You will submit Assignment Booklet 6B to your teacher at the end of this module.

Numbers in the News

Numbers are everywhere! Newspapers and magazines are full of stories and advertisements that show how numbers are used every day.

The following Scavenger Hunt asks you to look through newspapers and magazines for samples of number ideas like those you will be using in this module. Read through the list now and begin by collecting samples of the ideas you already understand. You may collect other samples as you learn about them in the module. Collect as many samples as you wish.

Scavenger Hunt

Cut out samples of each of the following number concepts from newspapers, flyers, and magazines.

1. predictions and forecasts
2. chance and uncertainty in everyday life
3. contests, games, or promotions

You will find further instructions for submitting your project in Assignment Booklet 6B.

Lesson 1

Describing the Chances

Have you ever noticed that if the past winter was unusually mild, many people in Canada fear the worst for the upcoming winter? They check almanacs and long-range forecasts to see if the most miserable weather in a hundred years is just around the corner. People are willing to bet that bad weather follows good. Do you think they are right?

The forecasts in almanacs are based on long-term trends and probabilities. Wouldn't you be surprised if almanacs didn't predict snow and a spell of cold weather for your region of Canada in January? In Canada's climate, it is not surprising that people plan their activities around the weather.

In this lesson you will review and extend your vocabulary for describing the chances of **events** happening. You will decide what events are possible in everyday situations and describe the likelihood that they will happen. You will compare situations to decide which one is more likely to happen and why you think that is so.

Activity 1

Today you will describe the likelihood of different events occurring.

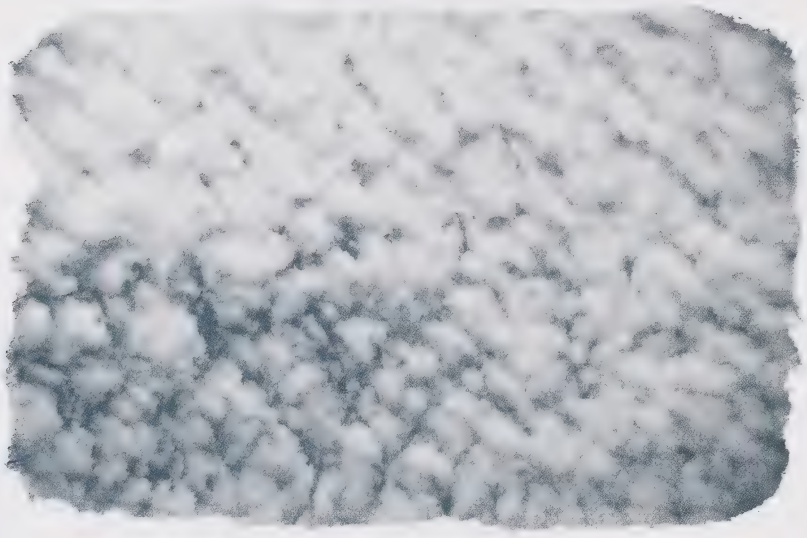
Being at sea during a storm can be very dangerous. My crew always looks for signs that will help us predict changes in the weather.
-Captain Quinn

In Captain Quinn's day, weather predictions were part of life at sea. The type of clouds, the colour of the sky, and the direction of the wind were all used to forecast the weather. This allowed sailors to properly set the sails and prepare for stormy seas. Over the years, these weather patterns were put into verses or short sayings.

Example

"Mackerel sky; not 24 hours dry."

This means there is always rain within 24 hours of a mackerel sky. (**Note:** A mackerel is a type of fish. A mackerel sky has small puffs of clouds that form long rows or waves that look like fish scales.)



Words such as *always*, *never*, *likely*, and *unlikely* are probability terms.

1. Use the probability term from the above list that best explains each of the following weather sayings.

a. "Red sky at night, sailors delight;
Red sky in the morning, sailors take warning."

A red sky at night means that it is _____ to rain the next day. A red sky in the morning means that it is _____ to rain that day.

b. "A change in the moon brings a change in the weather."

Weather _____ changes with the phases of the moon.

c. "If all the stars are out at night, it will be a nice day tomorrow."

A clear night is _____ to be followed by a pleasant day.

d. "When the wind is from the south, the rain's in its mouth."

A south wind is _____ to bring rain.

e. "With rain before wind, stays and topsails you must mind.
But with the wind before the rain, your topsails you may set again."

If it rains and then gets windy, it will _____ be rough sailing; but if it starts to rain on a windy day, the rain _____ will not last long.

f. "The wind in the west suits everyone best."

Westerly winds mean that there will _____ be good sailing.

g. "If clouds are gathering thick and fast,
Keep sharp look out for sail and mast."

A thick and fast cloud buildup is a signal that there will _____ be a storm.

h. "High clouds indicate fine weather will prevail."

High clouds mean that good weather will _____ continue.

i. "When clouds look like rocks and towers,
the earth will be refreshed by showers."

Rain clouds are _____ to have a rough and tall appearance.

j. "Mares' tails; storms and gales." (**Note:** Clouds known as mares' tails are fibrous, hooked-shaped clouds that are composed of ice.)

If clouds look like mares' tails, there will _____ be a storm.

k. "If it's foggy in the morning, then it'll be a sunny day."

It is _____ that a foggy morning will be followed by a sunny day.

l. "Rain before seven, clear by eleven."

If it starts to rain early in the morning, it will _____ last only a few hours.

m. "Seabirds, stay out from the land,
We won't have good weather while you're on the sand."

When seabirds stay by the shore, good weather is _____.

n. "Rainbow in the eastern sky, the morrow will be dry.
Rainbow in the west that gleams, rain falls in streams."

If you see a rainbow in the east, it is _____ that it will rain the next day; but a rainbow in the west will _____ mean heavy rain.

o. "If woolly fleeces spread the heavenly way,
Be sure no rain disturbs the summer day."

It _____ rains from soft, fluffy clouds.

2. Do you think these sayings are still useful today? Explain.

Check your answers on pages 62 and 63 in the Appendix.

For additional information on weather sayings. You may find the following Internet sites helpful:

- **http://classroomatsea.noaa.gov/shipops/rhymes.html**
- **http://www.meds-sdmm.dfo-mpo.gc.ca/cmos/weatherlore.html**
- **http://wilstar.com/skywatch.htm**

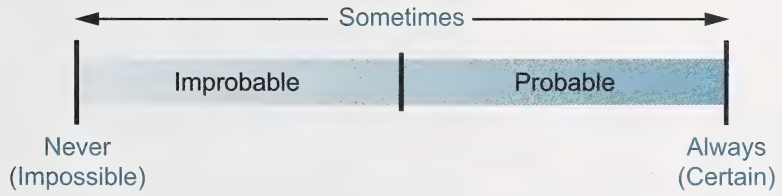
Activity 2

Today you will explore the vocabulary used to describe the likelihood of events occurring.

When describing or predicting events in everyday life, people may use words like *never* and *always*, even if they don't really mean what they say! For example, you might say, "I never get to stay up late at night," or " I always have to take out the garbage." It seems to be that way even though you know that there have been occasions when you have stayed up late or when somebody else took out the garbage.

When describing the probability of something, the word *never* means that it is **impossible** to happen, and the word *always* means that it is **certain** to happen. **Probable events** are likely to happen. **Improbable events** are unlikely to happen, although it is possible for them to take place.

You might picture impossible events at the left end of a number line, certain events at the right end, and events that **sometimes** happen as being somewhere in between.



Improbable events are left of centre; these events are unlikely to happen. Probable events are right of centre; these events are likely to happen.

1. Both probable and improbable events are possible. Which is more likely to happen, a probable event or an improbable event? Explain.

2. Turn to pages 182 and 183 in your textbook. Use the terms *always*, *never*, *sometimes*, *possible*, *impossible*, *probable*, *improbable*, *more likely*, or *less likely* to describe the events in each notebook. Explain your reasoning.

   Note: In some cases you will need to collect some data to have a better idea of how likely the event is.

Check your answers on pages 63 and 64 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Turn to page 80 of your Practice and Homework Book and answer questions 1 and 2. Discuss your answers with your home instructor.

Activity 3

Today you will use a number line to illustrate the likelihood of different events.

Some people are afraid of lightning. What are the chances of lightning hitting an object?

In the previous two activities you used words such as *always*, *never*, *likely*, and *unlikely* to describe the chance of an **event** occurring.

An event is a result that is of interest in a discussion of probability.

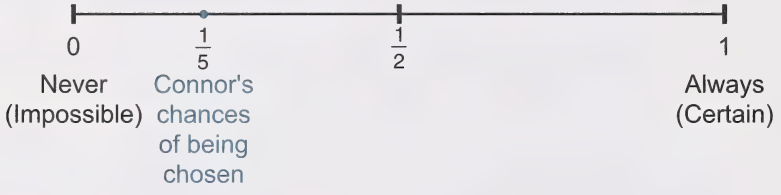
In this activity you will use a number to describe the probability of an event occurring.

The probability of an impossible event occurring is 0. The probability of a certain event occurring is 1. Fractions are used to describe the probability of an event that sometimes occurs.

Example

Kassidy and Connor belong to a book club that reads a different mystery novel each week. Each week, their teacher puts all the club members' names on separate slips of paper and puts them in her hat. She then chooses a name at random. The club member whose name she chooses reports to the entire class on the book the club has selected that week.

Kassidy has noticed that Connor reports to the class one-fifth of the time. She drew a number line to show the probability of Connor being chosen.



Kassidy put a dot one-fifth of the way between Never and Always.

Turn to pages 180 and 181 in your textbook. Use the pictures in Starting Out to answer questions 1 to 6.

1. a. Describe a situation when a light switch is very likely to be in the "on" position, as shown in the picture on page 180 of the textbook.

b. Describe a situation when the light switch could be in the "on" position shown, but you wouldn't expect it to be in that position.

2. Look at the picture of the baby on page 180 of the textbook. Draw a number line and mark and label points on it to show the chances of the baby being a boy or being a girl. Explain.

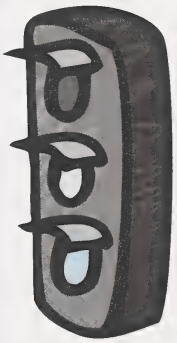
3. Look at the picture of the woman walking on page 180 of the textbook. She is approaching the entrance of a store.

   Describe the different things that might happen next, and give reasons why each may happen.

4. **a.** Look at the picture of the scoreboard on page 181 of the textbook and tell who won the football game.

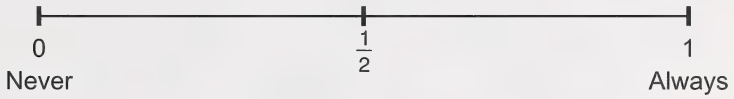
   b. Explain what things might affect who wins a football game.

5. **a.** When you look at a traffic light, what colour might it be?

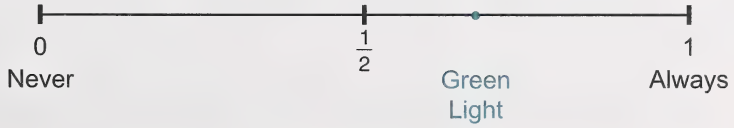


   b. Predict the chances of the traffic light being purple. Explain.

   c. Estimate, mark, and label a point on a number line like the one shown to predict the chances of the light being yellow. Explain.



   d. The following number line shows the chances of the light being green for the traffic on a freeway. Why might the light be set as shown?



e. If the light is set to green as in question 5.d., estimate, mark, and label a point on the line segment that shows the chances of the light being red. Explain.

Check your answers on pages 64 and 65 in the Appendix.

6. Draw a number line and mark and label a point on it to show how likely it is that a tree could grow money.

7. Draw a number line and mark and label a point on it to predict the chances of the sun rising in the east tomorrow.

The outcomes of some situations are the result of choice or skill, but the outcomes of other situations are **random**.

Random means occurring by chance.

8. a. Which of the situations in questions 1 to 7 can be influenced by choice or skill?

b. Which of the situations in questions 1 to 7 are random events?

9. Complete each sentence by using an example from your experience.

a. It is probable that _____.

b. It is improbable that _____.

c. It is impossible that _____.

d. It is certain that _____.

Check your answers on pages 65 and 66 in the Appendix.

Challenge Activity

A weather forecaster made the following statements:

- The chance of fog tomorrow is more likely than the chance of wind.
- The chance of snow tomorrow is less likely than the chance of rain.
- It is improbable that it will rain tomorrow.
- It is likely to be windy tomorrow.
- It is almost certain that the temperature will be greater than 0°C tomorrow.

Place the five weather conditions (fog, wind, snow, rain, 0°C) in order, from least likely to most likely, on a number line.

Check your answer on page 66 in the Appendix.

Conclusion

In this lesson you extended your vocabulary for describing the chances of events happening. You decided what outcomes are possible in everyday situations, and you described the likelihood that they could happen. You compared situations to decide which one is more likely to happen and why that is so.

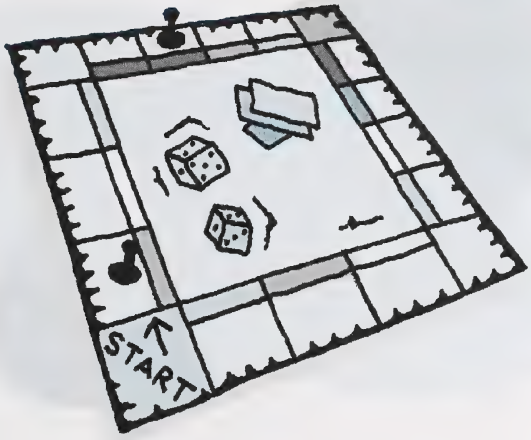
Weather forecasters frequently use the language and vocabulary of probability. What is the probability of rain tomorrow? Is it likely it will be sunny or is it more probable it will be cloudy? What are the chances it will be windy or calm? Check the forecast for your area on the radio or television or in the newspaper to find out!

Turn to Assignment Booklet 6A and complete the Lesson 1 Assignment.

Keep Assignment Booklet 6A until you have completed the entire booklet.

Lesson 2

Investigating Chance



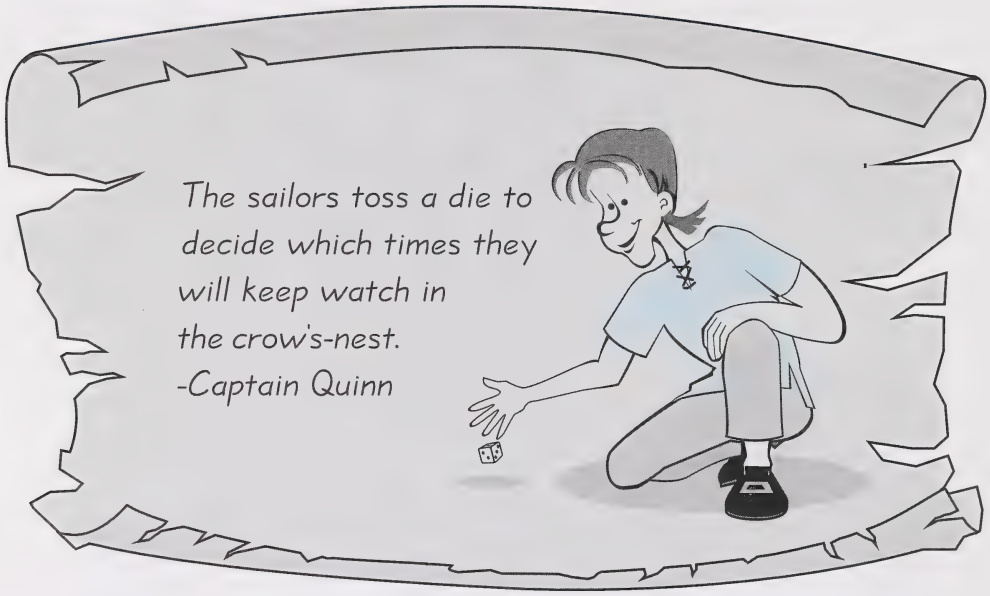
Do you like to play board games with your friends or family? Many board games involve rolling dice and moving a game piece a number of squares on the board equal to the sum of the dice. Part of the enjoyment in playing these games involves knowing your chances of rolling (or not rolling) a particular sum.

Games of chance using dice have a long history. Thousands of years ago, the Greeks and Romans (and before them, the people in ancient Egypt and Mesopotamia) played games with dice. Over the years, mathematicians have analysed these games, and their analyses have contributed to our knowledge of probability.

In this lesson you will use dice and spinners to conduct experiments and play games to help you discover more about probability. You will list the possible outcomes for these experiments, conduct these experiments, and then explain the results.

Activity 1

Today you will use dice to conduct probability experiments.

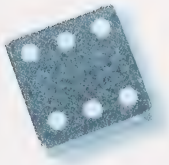


The following table shows the times of the watches when sailors are on duty in the crow's-nest:

Watch	Time Watch Begins
First	12:00 A.M. (midnight)
Second	4:00 A.M.
Third	8:00 A.M.
Fourth	12:00 P.M. (noon)
Fifth	4:00 P.M.
Sixth	8:00 P.M.

Take a die and look at it. Then answer the following questions.

1. When you toss a single die, what are all of the numbers that could come up?



2. Are some numbers more likely to come up than others? Explain.

3. Why did the sailors find a die useful to determine which watches they would keep?

Check your answers on page 66 in the Appendix.

One team of sailors was assigned to carry out all the different duties required for each watch. One member represented each team for the die toss, and they tossed in alphabetical order according to their first names. The first sailor tossed the die and got the watch number that came up. The remaining sailors, in turn, tossed the die as many times as necessary to get a watch number that had not yet been tossed.

4. The following table shows what happened when the sailors tossed the die.

Toss Order	Name	Numbers Tossed	Watch Number
1	Alex	4	Fourth
2	Bart	4, 2	Second
3	Casey	3	Third
4	Del	4, 3, 6	Sixth
5	Enja	2, 6, 4, 1	First
6	Fil	didn't have to toss	Fifth

   a. Would the first sailor to toss the die ever need to toss more than once? Explain.

   b. Would you expect that the fourth and fifth sailors would have to toss the die more than once? Explain.

   c. Find the total number of tosses the sailors made to determine all of their watches.

d. Why didn't Fil have to toss the die to determine which watch was his?

e. Is it likely or not likely that every sailor would have to toss the die just once?

5. To determine what watches the sailors might have had on the next voyage, you can carry out an experiment.

a. Predict the total number of times you will have to toss the die to determine all the watches. Explain.

b. Do you expect that any of the sailors will get the same watch on this voyage as they got on the last one?

c. Copy the following table. Then toss the die for each of the sailors and record your results.

Toss Order	Name	Numbers Tossed	Watch Number
1	Alex		
2	Bart		
3	Casey		
4	Del		
5	Enja		
6	Fil		

d. How many times in total did you have to toss the die?

e. Which, if any, sailors got the same watch on this voyage as the last one?

6. Some sailors preferred to work the day shifts; others preferred the night shifts.

a. If the sailors tossed in order of their ranks, from highest to lowest, would the higher-ranking sailors have a better chance than the lower-ranking sailors of getting shifts they liked? Explain.

b. If the sailors tossed in order of their ages, from oldest to youngest, would the older sailors have a better chance than the younger sailors of getting shifts they liked? Explain.

7. Bart said that the best result for him would be to get the 4:00 **P.M.** watch, and that the worst result for him would be to get the 4:00 **A.M.** watch.

a. Which number does Bart hope to toss, and what number does he not want to toss?

b. Is it more likely that the outcome will be best for Bart rather than worst?

Check your answers on pages 66 and 67 in the Appendix.

Activity 2

Today you will list all possible outcomes of an experiment involving a single event.

Sometimes a coin is flipped to decide which person will go first. For example, two tennis players could use a coin toss to decide who will serve first. Two golfers could use a coin toss to decide who will go up to the tee first. When someone flips a coin, there are two possible **outcomes**—"Heads" or "Tails."

Each of the outcomes of the coin toss is **equally likely** to occur.

In statistics, an outcome is the result of an action, such as flipping a coin. Equally likely outcomes have the same chance of occurring.

Not all actions have equally likely outcomes. For example, when a tack is tossed, there are two possible outcomes—point up and point down.

Top Point

Because the base of the tack is heavier than the pointed end, there is a greater chance of the tack landing point up. The outcomes are **not** equally likely to occur.

Kassidy and Conner decide to conduct a probability experiment by tossing a paper cup.

1. Turn to page 191 in your textbook. Look at the picture in question 2. It shows the three different ways a paper cup can land if it is tossed into the air.

a. Describe each of the possible outcomes.

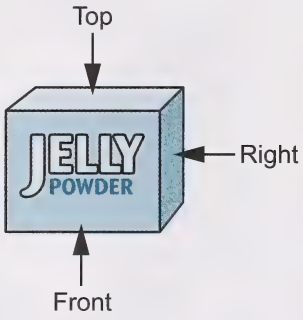
b. Predict the likelihood of each of the three outcomes. Explain.

c. Carry out an experiment to check your prediction by tossing the paper cup 20 times. Make a tally chart and use it to record your results.

d. Was your prediction correct?

Check your answers on page 68 in the Appendix.

2. Imagine that you had a box like the one below, and you tossed it high into the air.



a. Describe all of the different ways the box can land. Explain.

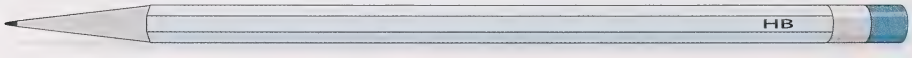
b. Which outcomes are equally likely to happen? Explain.

c. Which outcome do you predict is more likely to happen than any other outcome? Explain.

d. Carry out an experiment to check your prediction by tossing such a box 20 times. Make a tally chart and use it to record your results.

e. Was your prediction correct?

3. Use a hexagonal pencil that has HB (or some other writing) stamped along one face, like the one shown.



a. Use probability terms to describe the likelihood that the pencil will stop with the printed face exactly on top if you roll it across a table or floor.

b. If you roll the pencil 30 times, predict how many times it will stop with the printed face exactly on top. Explain.

c. Carry out the experiment described in question 3.b. Make a tally chart and use it to record your results.

d. How close was your prediction? Explain.

4. Kassidy said, "The shape of a die looks more like the shape of a jelly powder box than the shape of a hexagonal pencil, but the outcomes of tossing a die are more like the outcomes of rolling a pencil than those of tossing a jelly powder box." Explain why she thought this.

Check your answers on pages 68 to 70 in the Appendix.

Activity 3

Today you will use spinners to conduct probability experiments. You will also consider fairness.

Spinners are used in many board games.

Turn to page 184 in your textbook. Use the spinners shown in Spinner Experiments to answer questions 1 to 5.

1. Look at the top-left spinner on page 184.

   a. About what fraction of the spinner is green? About what fraction of it is red?

   b. If you twirled this spinner 24 times, about how many times would you expect the pointer to stop on red? Explain.

2. Look at the top-right spinner on page 184.

   a. Compare the chance of the pointer stopping on the hearts with the chance of it stopping on the stripes. Explain.

   b. If you twirled this spinner 50 times, about how many times would you expect the pointer to land on the hearts? Explain.

3. Look at the bottom-left spinner. For each time the pointer stops on white, how many times would you expect it to stop on blue? Explain.

4. Look at the bottom-right spinner. Dianne said that the chance of the pointer stopping on the stars and the chance of tossing a 5 on a regular die are equally likely. Do you agree with Dianne? Explain.

Check your answers on page 70 in the Appendix.

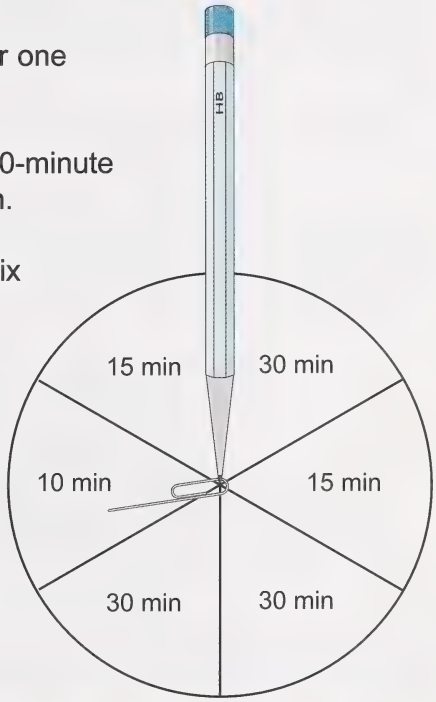
Turn to page 185 in your textbook. Use the spinners shown to answer questions 6 to 8.

5. Look at the top-left spinner on page 185.

a. How likely are you to spin a 30-minute recess compared to either one of the other two times? Explain.

b. Is it more likely that you will spin a 30-minute recess than a different time? Explain.

c. Find the page of dials in the Appendix and use the top one to conduct an experiment. Make sure that your page is flat. Put the tip of your pencil on the centre of the circle so it holds a paper clip for a spinner. Spin the paper clip by flicking it with your finger enough times (at least 20) for you to decide which outcome is most likely. Make a tally chart like the following and use it to record your results.



Outcomes	Tallies	Frequency
10 Minutes		
15 Minutes		
30 Minutes		
	Total	

d. Were the results of your experiment as you expected?

Check your answers on pages 70 and 71 in the Appendix.

6. Look at the top-right spinner on page 185.

   a. How likely are you to spin popcorn compared to either one of the other two treats? Explain.

   b. Is it more likely that you will spin popcorn than a different treat? Explain.

   c. Find the page of dials on the Appendix and use the bottom one to conduct an experiment. Make sure that this page is flat. Put the tip of your pencil on the centre of the circle so it holds a paper clip for a spinner. Spin the paper clip by flicking it with your finger enough times (at least 20) for you to decide which outcome is most likely. Make a tally chart like the following and use it to record your results.

Outcomes	Tallies	Frequency
Popcorn		
Muffins		
Cookies		
	Total	

   d. Were the results of the experiment as you expected?

7. Look at the bottom spinner on page 185.

   a. Predict your chance of spinning 30 minutes of homework. Explain.

   b. About what fraction of the spinner shows 60 minutes?

   c. Which spinner parts could you change to 60 minutes so that the spinner would likely stop on 60 minutes half of the time? Explain.

Check your answers on pages 71 and 72 in the Appendix.

So far in this module, you have found the outcomes of tossing a die, a coin, a tack, a cup, and a box that is not a cube. You found the outcomes of rolling a pencil and twirling a spinner.

In some situations, the outcomes are equally likely to occur. These situations are **fair**.

Examples include tossing a die, tossing a coin, rolling a pencil, or twirling the spinner on a dial that is equally divided.

In other situations, one outcome is likely to occur more often than another. These situations are **unfair**.

Examples include tossing a tack, tossing a glass, tossing a box that is not a cube, or twirling the spinner on a dial that is not equally divided.

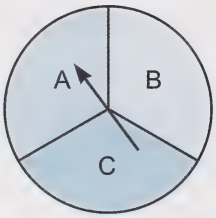
8. Look at the centre spinner on page 184. Billie asked Barrie to play a game with this spinner. Billie said that every time the pointer stops on green, she will get 1 point; and every time it stops on blue, Barrie will get 2 points. Is this fair? Explain.

Check your answer on page 72 in the Appendix.

Rules are sometimes made to ensure that an event is fair and not influenced by outside factors such as the experience or the skill of the participants.

For example, when rolling a die, the participants are expected to shake the die vigorously. Sometimes the participants must shake the die in a cup.

9. Write a rule to ensure that twirling the spinner on this dial is fair.



Check your answer on page 72 in the Appendix.

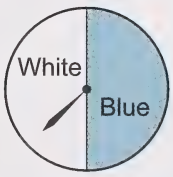
Sharing Time

Now it's time to show your home instructor what you have been learning.

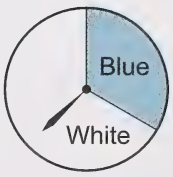
Complete the question on page 82 of your Practice and Homework Book. Discuss your results with your home instructor.

Challenge Activity

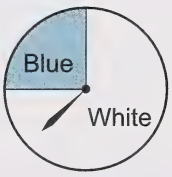
Jim is playing a board game with a spinner. If the pointer stops on white, he moves ahead 5 spaces. If the pointer stops on blue, he moves back 2 spaces. He wants to win a sack of gold that is 16 spaces ahead of where his game piece is now. He wants to use the least number of spins possible so that he can get to the sack of gold before his opponent does. Which spinner will give Jim his best chance of getting the sack of gold? Explain.



Spinner 1



Spinner 2



Spinner 3

Check your answer on page 72 in the Appendix.

Conclusion

In this lesson you conducted experiments and played games. You listed all the possible outcomes for an experiment, conducted the experiment, and then explained your results. You decided whether a die or spinner was used in a fair way. You made spinners, predicted the outcomes for using them, and then checked your predictions by conducting experiments.

Board games and dice have been played and enjoyed by people of all cultures. Dice have been found that date back to the Pharaohs of ancient Egypt, over three thousand years ago. Why do you think games involving an element of chance have been so popular?

Turn to Assignment Booklet 6A and complete the Lesson 2 Assignment.

When you are done, send Assignment Booklet 6A to your distance learning teacher to be marked.

Lesson 3

Chance—Past and Present

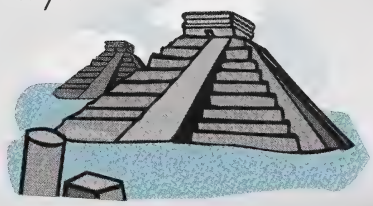
The people who first populated the Americas also invented and enjoyed playing games of chance. For example, the Mayan civilization that flourished for over three thousand years between 1500 B.C. to 1700 A.D. is remembered not only for its pyramids and temples, calendars, astronomy, and mathematics, but also for the music, dress, and games of its people. It is not surprising that this culture that depended so heavily on the cultivation of corn developed games of chance using kernels of corn as markers.

In this lesson you will see how probability has always been part of everyday life. You will play an ancient Mayan game of chance, collect data from **samples**, and use your results to make predictions about the **populations** from which you took the samples.

Activity 1

Today you will explore the concept of fairness in games of chance.

On one of our trips to Central America, some of my crew were most intrigued by a game board they found scratched into the stone floor of a Mayan building.
-Captain Quinn



Games that are based on chance are part of almost every culture. The ancient Mayans played one such game called Bul (pronounced *Bool*). The Mayans didn't have dice, so they tossed four flat kernels of corn with a black mark burned on one side of each kernel. After tossing the corn, they counted the number of burned sides showing. The following table shows how counts of 1, 2, 3, 4, and 5 are scored.

Toss					
Count	1	2	3	4	5

The game of Bul is still played today in Central American countries, such as Honduras. Each player uses five game pieces; the pieces could be pebbles, shells, dried beans, or seeds. The game "board" is made with 15 objects placed in a row or 15 marks etched in the ground. The 14 spaces between them is where the game pieces are moved.

Two players start at opposite ends of the board. The game begins with each player placing a game piece at his or her end. Each player gets two throws of corn in a row and moves the number of spaces indicated after each throw. Each player has only one game piece for each round of play.

When a game piece gets to the end of the board, it is re-entered at the end where it started, as if the board were circular. A round of play ends when one player captures an opponent's piece by landing on the space occupied by that piece. Another round then starts, with each player placing a new game piece at his or her end. This continues until one player wins the game by capturing all of the opponent's game pieces.

You can find out more about the game Bul on the Internet at the following website:

http://www.halfmoon.org/bul.html

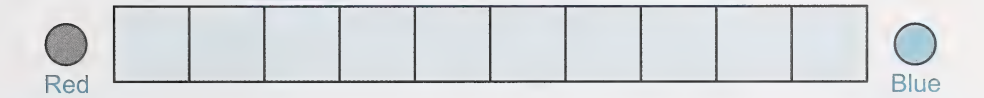
1. **a.** Do you think Bul is a fair game? Explain.
   b. How can you test your answer?

Check your answers on page 73 in the Appendix.

To test the fairness of Bul, you can play a modified version of the game. Here are the rules for a modified game of Bul:

- Use a single die and two paper game pieces. Colour one game piece red and the other blue.
- The playing board has only ten spaces.
- Take one turn at a time, tossing the die and moving your game piece (blue moves left and red moves right) as many spaces as the die shows. Blue tosses first to start every round.
- Pieces that go past the end of the game board start back at the end where they started.
- A piece is captured when the other player lands on it. This is the end of that round.

GAME BOARD FOR MODIFIED BUL



2. Play 20 rounds of the modified game, using the game board shown. Remember, blue tosses first to start every round. You may ask someone to play with you or you may play it alone. Record your results in a table like the one that follows.

Colour	Who Won the Round? (tallies)	Total Rounds Won
Blue		
Red		

3. Based on your results, do you think this is a fair game?

4. What is the fewest number of tosses needed to complete a round?

5. What is the greatest number of tosses that may be needed to complete a round?

6. Julio enjoyed playing modified Bul at home. When he showed the game to his friend, Breanne, she said she didn't think she would beat him very often. "You've played it so many times that you must be very good at it." What would you tell Breanne?

Check your answers on page 73 in the Appendix.

Sharing Time

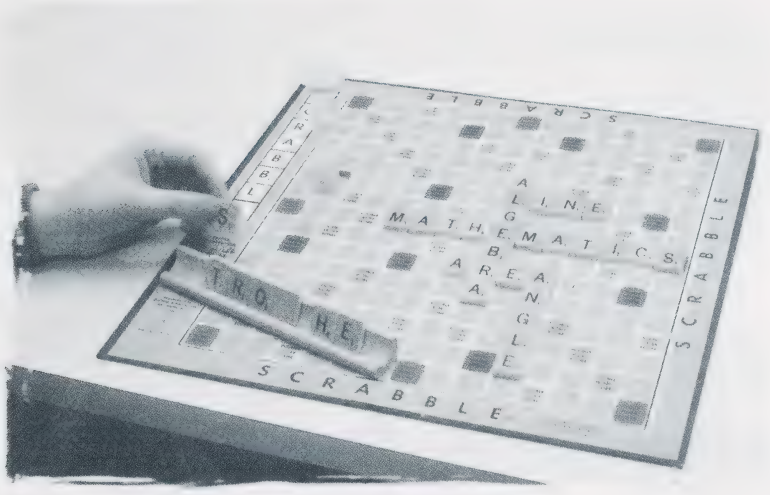
Now it's time for you to show your home instructor what you have been learning.

Play a game such as Snakes and Ladders, Go Fish, or Checkers with your home instructor.

Discuss the importance of skill and chance in the game.

Activity 2

Today you will conduct a probability experiment with letters.



Have you ever played Scrabble®? Scrabble® is a popular crossword game in which players take turns spelling words with letter tiles. Did you ever wonder how the manufacturers of Scrabble® decided how many tiles to make for each letter of the alphabet?

1. Which five letters do you think are used most often in the English language?

2. Which letter do you think is used least often in the English language?

Check your answers on pages 73 and 74 in the Appendix.

You can test your prediction by choosing a **sample** of writing in the English language, counting the number of times each letter is used, and then listing the letters in order of frequency.

A sample is a part of the **population** from which information is gathered. The population is the entire group of people or things for which information is needed.

3. Turn to page 187 in your textbook.

   a. How many letters do you think there are in the yellow box on page 187?

   b. Do you think this would be a suitable sample to use to check the predictions you made in questions 1 and 2? Explain.

   c. Use a table like the following to tally the frequencies of all the letters inside the yellow box on page 187. (If possible, work with your home instructor or someone else who can read the letters to you as you record the tallies.)

	Frequency of Letters on Page 187 of Textbook	Total
A		
B		
C		
X		
Y		
Z		
	Total	

4. a. Write the letters you tallied in order from greatest frequency to least frequency.

   b. How do the resulting frequencies compare with your predictions?

Check your answers on pages 74 and 75 in the Appendix.

5. **a.** How do you think the letter frequencies in a sample of English writing taken from a book you are currently reading, or from a newspaper article, would compare with the letter frequencies in the textbook sample?

   b. Choose a sample of English writing with about the same number of letters as the textbook sample. Use a table like the one you used in question 3.c. to tally the letter frequencies for your chosen sample. (If possible, work with your home instructor or someone else who can read the letters to you as you record the tallies.)

6. **a.** Write the letters in your sample in order from greatest frequency to least frequency.

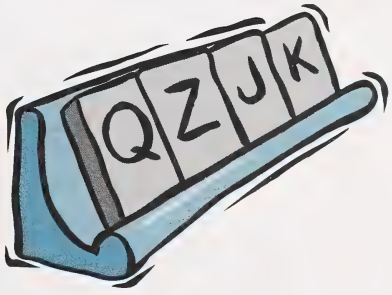
   b. How do the resulting frequencies compare with the textbook sample?

Check your answers on pages 75 to 77 in the Appendix.

7. **a.** Suppose you were making your own crossword board game that had 98 letter tiles. In a table like the one below, show how many of the 98 tiles you would use for each letter. Then write the letters in order from the greatest frequency to the least frequency.

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Number													

   b. Scrabble uses 98 letter tiles (plus 2 blank tiles). Compare your tiles with the 98 Scrabble tiles. Explain how the two sets of tiles are the same and how they are different.



Check your answers on pages 77 and 78 in the Appendix.

Activity 3

Today you will conduct a probability experiment with M&M's® candies.

If you enjoy M&M's® candies, you probably have a favourite colour. Is it blue or red? You may be curious about your chances of getting your favourite colour when you buy a package of M&M's®.

To investigate, you will need a 48-g package and a 159-g package of milk chocolate M&M's® as samples.

1. What is the population for which you are collecting data?

2. What colours are there for milk chocolate M&M's® candies?

3. **a.** Predict which colour you think will have the greatest frequency.
   b. Predict which colour you think will have the least frequency.

4. Carefully empty your package of milk chocolate M&M's®, and use tables like the ones below to record your data.

Colours of M&M's®	Number of Each Colour in My 48-g Sample (tallies)	Total Frequency
Blue		
Brown		
Green		
Orange		
Red		
Yellow		

Colours of M&M's®	Number of Each Colour in My 159-g Sample (tallies)	Total Frequency
Blue		
Brown		
Green		
Orange		
Red		
Yellow		

5. **a.** Was your prediction in question 3.a. about the most frequent colour correct?

   b. Was your prediction in question 3.b. about least frequent colour correct?

Check your answers on pages 78 and 79 in the Appendix.

6. The following table indicates the fraction of each colour of milk chocolate M&M's® that the company makes. Use these decimal fractions to calculate the number of each colour expected.

Colours of M&M's®	Fraction of Each Colour	Number Expected in a 48-g Package (with 60 M&M's®)	Number Expected in a 159-g Package (with 180 M&M's®)
Blue	0.1		
Brown	0.3		
Green	0.1		
Orange	0.1		
Red	0.2		
Yellow	0.2		

7. Compare the results for your samples with the completed table.

   a. Explain how the results for your 48-g sample compare.
   b. Explain how the results for your 159-g sample compare.

8. A larger sample usually predicts what a population is like better than a smaller sample does. Is this true for your two samples? If not, why do you think this is so.

9. Why doesn't every package of milk chocolate M&M's® in a particular size have the same number of each colour?

10. Some colours used for M&M's® have been changed over the years. In 1941, when M&M's® were first introduced, the colours were brown, red, yellow, green, orange, and violet. The violet colour was replaced first by tan in 1949, and again by blue in 1995.

    Why do you think they made the colour changes, even though the candies all taste the same?

Check your answers on pages 79 and 80 in the Appendix.

To discover some interesting facts about M&M's®, you may wish to visit the following website:

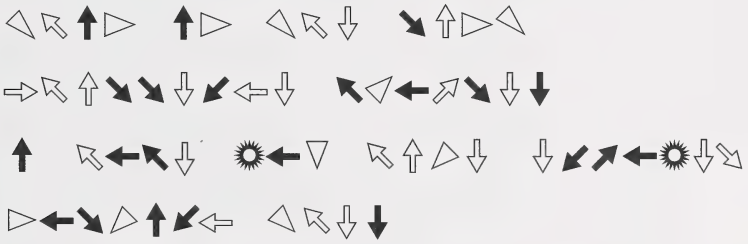
http://www.m-ms.com

Challenge Activity

1. In the following message, each letter of the alphabet has been replaced with a different letter. Find the message.

   XUW XKGIJEW YZXU IFZDC P AGNW EZQW XUZF GDW ZF XUPX ZX ZF WPFR LGK GXUWK TWGTEW XG ZL XUWR QDGY PJGIX EWXXWK LKWBIWDAZWF.

2. In the following message, each letter of the alphabet has been replaced with a different symbol. Find the message.



Check your answers on page 80 in the Appendix.

The puzzles you have just completed are called **cryptograms**. If you would like to make a cryptogram of your own or solve another cryptogram, you will find the following website helpful:

http://www.wordles.com

Conclusion

In this lesson you saw how probability has always been part of everyday life. You played an ancient game of chance, collected data from samples, and used your results to make predictions about the populations from which you took the samples.

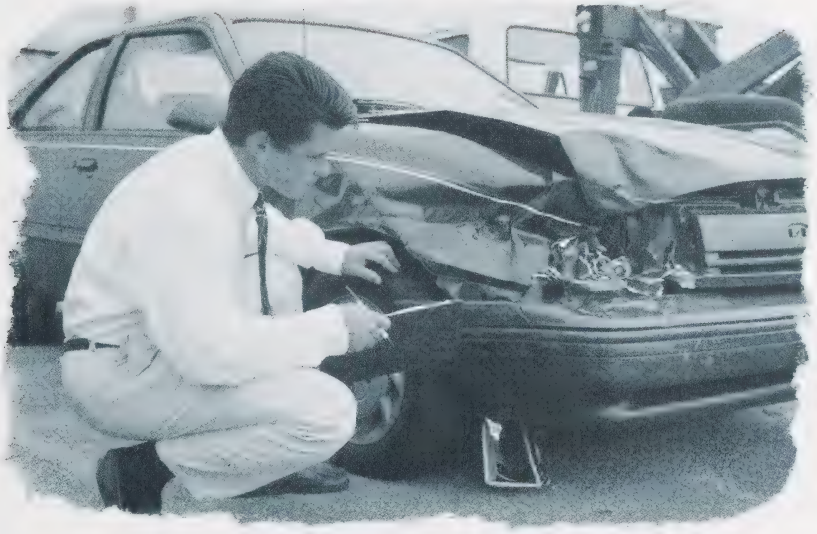
Games of chance have a long history in the Americas, from the ancient game of Bul to modern games, such as Monopoly®. Do you have a favourite game that involves the element of chance or luck? Do you think people will still be playing this game three thousand years from now?

Turn to Assignment Booklet 6B and complete the Lesson 3 Assignment.

Keep Assignment Booklet 6B until you have completed the entire booklet.

Module Summary

In Module 6 you saw how probability has always been part of everyday life. You reviewed and extended your vocabulary by describing the outcomes of everyday events and by comparing situations to decide which one is more likely to happen. You played games using spinners, dice, and other objects, and you decided if the games were fair. You listed all the possible outcomes for experiments, conducted them, and explained the results. You collected data from samples and used your results to make predictions about the populations from which you took the samples.



Many professionals use the principles of probability in their daily work. Insurance brokers, meteorologists, agricultural forecasters, lottery officials, and sports broadcasters are just a few. It is very likely that you will use these principles in the career you choose.

Turn to Assignment Booklet 6B and complete the Numbers in the News project.

When you are done, send Assignment Booklet 6B to your distance learning teacher to be marked.

Keystrokes

Take out your calculator and complete the following exercise. It will help you review some of the ideas you have learned in Module 6.

Funky Feature: Take-Away Toss

Here's a calculator game you can play with a partner or on your own. If you play with a partner, you'll each need a calculator.

- Each player enters the number 123456 into his or her calculator.
- Toss a regular die to see who goes first. (The greatest number wins.)
- On each turn, a player tosses the die and "erases" the number tossed from his or her calculator display. (In Module 3, you learned how you can erase a number from your calculator display by knowing its place value.)
- If you toss a number that has already been erased, you must wait for your next turn to toss again.
- Keep taking turns tossing the die and erasing digits until one player's display reads zero. That player wins.

If you play the game on your own, keep track of how many tosses you make. Your goal is to erase all your digits in ten tosses or less.

1. If you are playing with a partner, is this a fair game? Explain.

2. a. Keep track of how many tosses are needed each time to erase all six digits (whether you are playing with a partner or by yourself). Use a table like the following to record your results for ten games.

Game	1	2	3	4	5	6	7	8	9	10
Number of Tosses Needed										

b. What was the least number of tosses that you needed?

c. What was the greatest number of tosses that you needed?

d. What is the least number of tosses you could make to win the game? Explain.

e. What is the greatest number of tosses you might need to win the game? Explain.

f. Do you think the goal of ten tosses for a solitaire game is reasonable? Explain.

Check your answers on pages 80 and 81 in the Appendix.

This review will help you apply what you learned in Module 6 and prepare for the final test. Discuss with your home instructor when you should begin the Review and how much of the Review you should complete.

1. Use probability terms to explain the meaning of the following sayings.

a. When a halo rings the moon or sun,
Rain's approaching on the run.

b. Ice in November to bear a duck,
The rest of the winter'll be slush and muck.

c. The moon her face be red, of water she speaks.

2. Turn to page 223 of your textbook. Do question 4 of Skill Bank Looking Back.

3. Turn to page 196 of your textbook. Do question 2 of Skill Bank from This Unit.

4. Compare the probabilities of each of the following pairs of events.

   a. In Alberta, it will snow in June.
      In Alberta, it will snow in November.

   b. I will be awake at 3:00 A.M. tomorrow.
      I will be awake at 3:00 P.M. tomorrow.

   c. The next person to enter a store will be male.
      The next person to enter a store will be female.

If you need help with questions 1 to 4, look back at Lesson 1, where you learned about describing chance. If you feel you need more practice, do questions 5 and 6.

5. Write a sentence that describes the likelihood of each of the following events.

   a. You will travel to the moon tomorrow.
   b. You will have lunch tomorrow.
   c. You will find a penny today.
   d. You will read a sentence today.

6. Finish each of the following sentences.

   a. Milk will likely go sour if _____.
   b. It is certain that tomorrow _____.
   c. It is improbable that I will be outside _____.
   d. Cats never _____.

Check your answers on pages 81 and 82 in the Appendix.

7. Turn to page 196 of your textbook. Do question 3 of Skill Bank from This Unit.

8. Turn to page 186 of your textbook. Do questions 1 to 4 of Practise Your Skills.

9. Turn to page 196 of your textbook. Do question 1 of Skill Bank from This Unit.

10. Turn to page 243 of your textbook. Do question 1 of Skill Bank Looking Back.

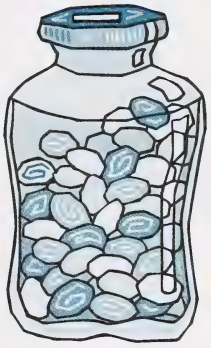
If you need help with questions 7 to 10, look back at Lesson 2, where you learned about investigating chance. If you feel you need more practice, do questions 11, 12, and 13.

11. Turn to page 186 of your textbook. Do question 1 of On Your Own.

12. Turn to page 223 of your textbook. Do question 3 of Skill Bank Looking Back.

13. Turn to page 190 of your textbook. Do question 1 of Practise Your Skills.

Check your answers on pages 82 to 84 in the Appendix.



14. Maebelle wondered if there was the same number of each colour of jellybean in the large bin of bulk jellybeans at the grocery store. She bought a scoop of them and counted them at home. Her results are shown in the following table.

Colour	Red	Yellow	Pink	Orange	Black	Green	White	Purple
Number	17	16	7	21	15	10	8	6

Explain why you agree or disagree with each of Maebelle's conclusions.

a. It is improbable that the company makes the same number of each colour.

b. The company likely makes more red jellybeans than any other particular colour.

c. It is uncertain for which colour the company makes the least number of jellybeans.

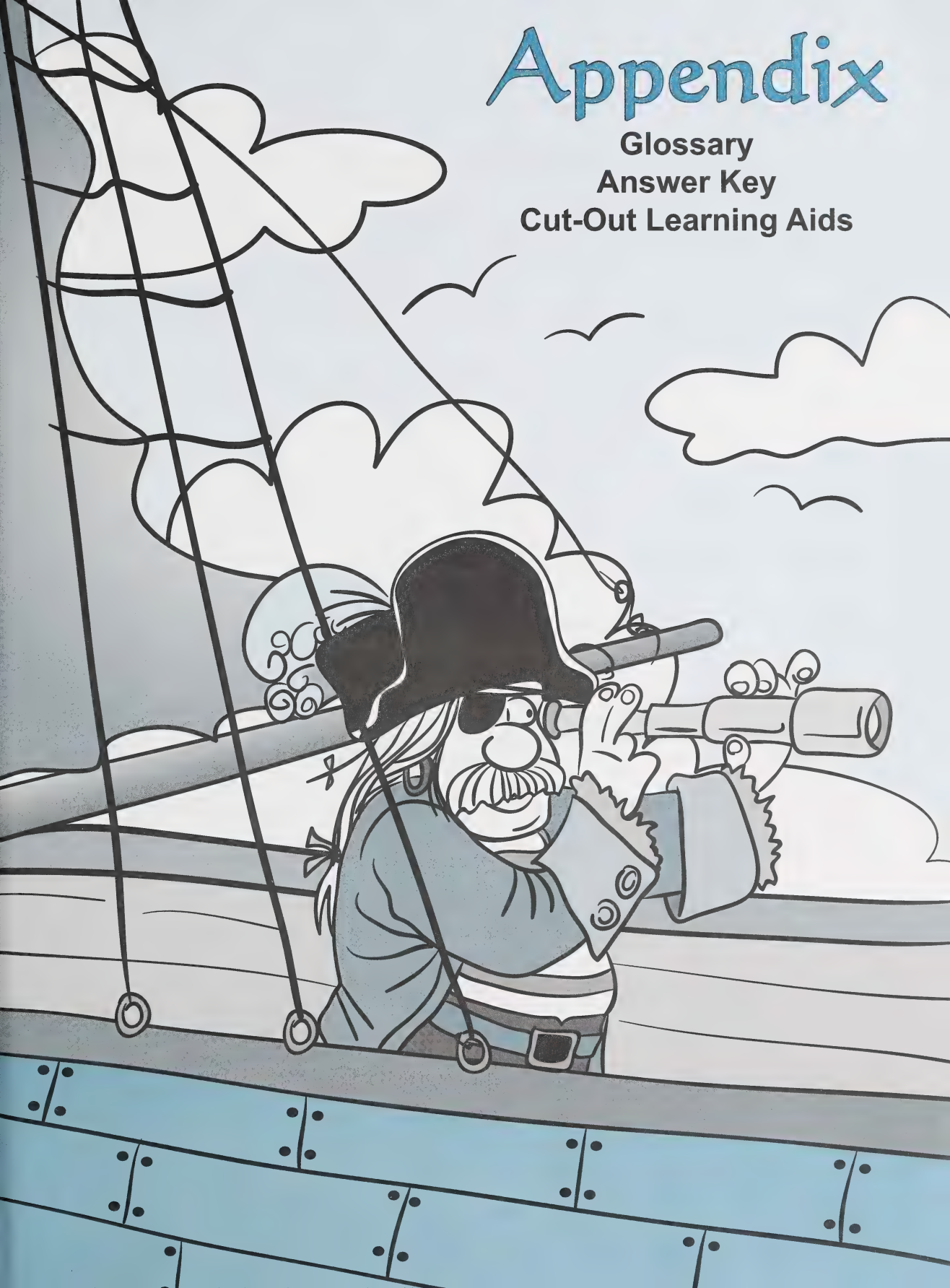
If you need help with question 14, look back at Lesson 3, where you learned how chance is used in everyday life. If you feel you need more practice, do question 15.

15. To investigate which letter is most frequently used in the English language, three friends each used a different sample of written words, as described below:

- Harley opened up a novel and used the page where it opened.
- Van opened up a dictionary and used the page where it opened.
- Sooni opened up a book about zebras in the zoo and used the page where it opened.

Which of the three samples most likely represents the population being investigated? Explain.

Check your answers on page 84 in the Appendix.



Appendix

Glossary
Answer Key
Cut-Out Learning Aids

Glossary

event: a result that is of interest in a discussion of probability

fair: when outcomes are equally likely to happen

improbable event: an event that is unlikely to occur

population: an entire group of people or things for which information is needed

probable event: an event that is likely to occur

random: occurring by chance

sample: a part of a population from which information is gathered

unfair: when one outcome is more likely to happen than another

Answer Key

Lesson 1: Describing the Chances

Activity 1

1. **a.** A red sky at night means that it is unlikely to rain the next day. A red sky in the morning means that it is likely to rain that day.

   b. Weather always changes with the phases of the moon.

   c. A clear night is likely to be followed by a pleasant day.

   d. A south wind is likely to bring rain.

   e. If it rains and then gets windy, it will likely be rough sailing; but if it starts to rain on a windy day, the rain likely will not last long.

   f. Westerly winds mean that there will likely be good sailing.

   g. A thick and fast cloud buildup is a signal that there will likely be a storm.

   h. High clouds mean that good weather will likely continue.

i. Rain clouds are likely to have a rough and tall appearance.

j. If clouds look like mares' tails, there will likely be a storm.

k. It is likely that a foggy morning will be followed by a sunny day.

l. If it starts to rain early in the morning, it will likely last only a few hours.

m. When seabirds stay by the shore, good weather is unlikely.

n. If you see a rainbow in the east, it is unlikely that it will rain the next day; but a rainbow in the west will likely mean heavy rain.

o. It never rains from soft, fluffy clouds.

2. These sayings are still useful today because they are based on weather patterns over many years of past observations.

Activity 2

1. A probable event is more likely to happen than an improbable event. A probable event is likely to happen and an improbable event is unlikely to happen.

2. • An ice cube on my desk will **always** melt. In the winter, an ice cube is **more likely** to melt faster on my desk than it will outside. On a summer day, an ice cube is **less likely** to melt faster on my desk than it will outside.

   • Whether it is **more likely** to rain than it is to snow this week depends on the temperature. It is **more likely** to rain when temperatures are warmer than 0°C. It is **more likely** to snow when temperatures are colder than 0°C.

   • In Alberta, the sun **always** sets before 7 **P.M.** in the winter, and it **always** rises before 7 **A.M.** in the summer.

   • My birthday is **always** on the same date each year. My birthday is **never** on the same day of the week for two years in a row.

- If only Grade 5 students are included, it is **impossible** for someone in the class to be 50 years old. Therefore, the Grade 5 students in the class will **always** be younger than 50 years old.

- It is **probable** that I can hold my breath for 15 seconds, 30 seconds, or 60 seconds. It is **less likely** that I can hold my breath for 60 seconds than I can for 15 seconds or for 30 seconds. It is **more likely** that I can hold my breath for 15 seconds than for 30 seconds.

- If there are mostly Grade 5 students in the room, it is **probable** that lots of them are wearing running shoes. It is **less likely** that they are wearing dress shoes or hiking boots.

- It is **probable** that someone in the class will celebrate a birthday this month. It is **possible**, but **less likely**, that someone in the class will celebrate a birthday this week.

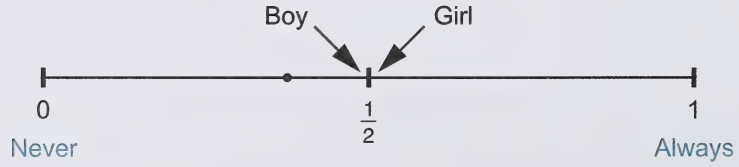
- It is **probable** that in any class you pick there will be two people with the same birthday.

Activity 3

1. **a.** The light switch is very likely to be in the position shown if people are in the room and the light is switched on.

   b. The light switch might be in the position shown if nobody is in the room, if somebody fell asleep at night with the light on, or if the light bulb burned out after the light was turned on.

2. About half the babies born are girls and about half the babies born are boys, so the middle of the line can be marked to stand for both.



3. The woman may open the door and go in if she wants to buy something from the store or if she works there. She might walk by the store if she is an a hurry or if she simply doesn't need to shop for anything.

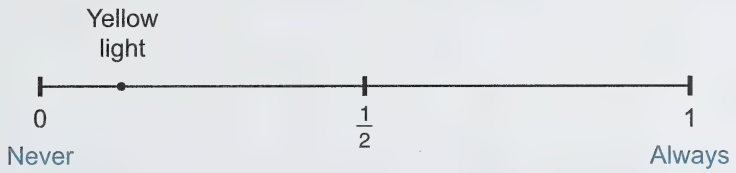
4. **a.** The visitors won the football game.

   b. Chances of winning the game may be affected by the skill of the players, luck, injuries, or penalties.

5. **a.** A traffic light might be red, green, or yellow when you look at it.

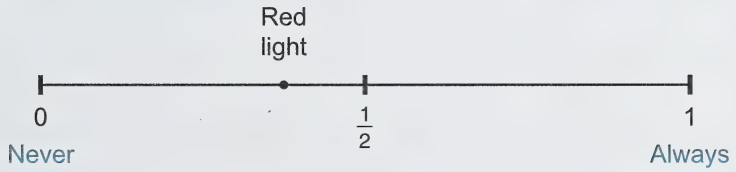
   b. The traffic light is never purple. Purple is not a colour used for traffic lights.

   c. The light is yellow for just a short time, so the point is close to Never.



   d. The light might be set green as shown because the freeway traffic is much heavier than the traffic on the intersecting road.

   e. The light must be red for less than half of the time if it is green for more than half of the time. (The total of the probabilities for yellow, green, and red must equal 1.)



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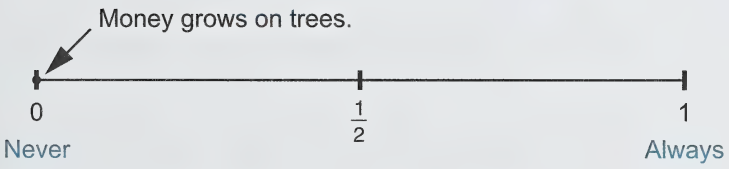

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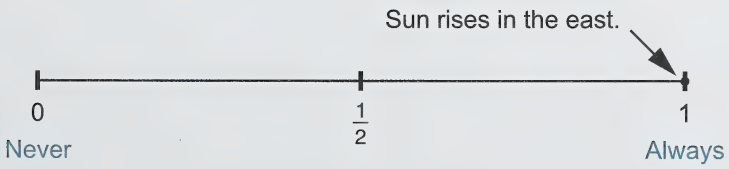
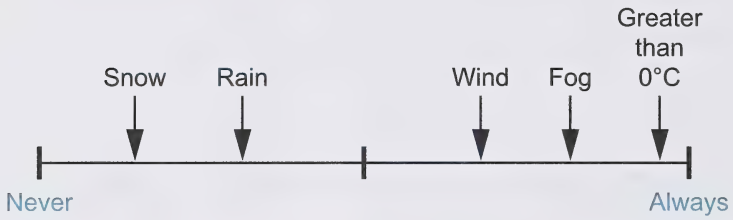


8. **a.** The light switch and the woman by the store can be influenced by choice. The football game can be influenced by skill.

   b. The traffic lights and the baby are random events.

9. **a.** It is probable that I will eat breakfast at home tomorrow.
   b. It is improbable that I will eat breakfast at a restaurant tomorrow.
   c. It is impossible that my dog will speak English.
   d. It is certain that I will be older tomorrow than I am now.

Challenge Activity



Lesson 2: Investigating Chance

Activity 1

1. When you toss a single die, 1, 2, 3, 4, 5, or 6 could come up.

2. No, all numbers are equally likely to be rolled because a die has only one of each number on it.

3. The sailors found a die useful to determine which watches they would keep because there were six different watches and there are six different numbers on a die.

4. **a.** The first sailor to toss the die would never need to toss more than once because no number would have come up before his throw.

   b. The fourth and fifth sailors to toss the die would probably have to toss it more than once because it is likely that they would toss a number that someone had already tossed.

c. Eleven tosses were needed to determine all of the watches.

d. Fil didn't have to toss the die because he was the last sailor and the fifth watch was the only one not yet taken.

e. Although it is possible, it is not likely for every sailor on his or her first try to toss a number that no one else had yet tossed.

5. Answers will vary. Sample answers are given.

a. I think I will have to toss the die about 10 to 12 times. In question 4.c., the sailors had to toss the die 11 times.

b. I won't be surprised if one of the sailors gets the same watch on both voyages.

c.

Toss Order	Name	Numbers Tossed	Watch Number
1	Alex	3	Third
2	Bart	6	Sixth
3	Casey	4	Fourth
4	Del	5	Fifth
5	Enja	4, 4, 5, 4, 4, 1	First
6	Fil	didn't have to toss	Second

d. I had to toss the die ten times.

e. Enja got the first watch on both voyages.

6. a. The higher-ranking sailors wouldn't have a better chance of getting shifts they liked. It is equally likely for any of the numbers to come up on any toss.

b. The older sailors wouldn't have a better chance of getting shifts they liked. It is equally likely for any of the numbers to come up on any toss.

7. a. Bart hopes to toss a 5. He does not want to toss a 2.
b. No, Bart's chances are equally likely for getting his best result or his worst result.

Activity 2

1. **a.** The cup can land with its open end up, on its side, or with its open end down.

   b. The cup will land on its side most often because the curved surface of the cup has a greater area than the top or the bottom. The cup will land with its open end down more often than with its open end up because the top circle has a greater area than the bottom circle.

   c. Answers will vary. Sample data are provided.

Outcomes	Tallies	Frequency
Open End Up	|	1
On Its Side	𝍸 𝍸 𝍸 ||	17
Open End Down	||	2
	Total	20

   d. Based on the sample data, the predictions were correct. The cup landed on its side much more often than it landed with its open end up or its open end down.

2. **a.** There are six possible outcomes. The box can land on any of its six faces: the front face, the back face, the top face, the bottom face, the right face, or the left face.

   b. It's equally likely for the box to land on its front face or its back face because they are the same size.

   It's equally likely for the box to land on its top face or its bottom face because they are the same size.

   It's equally likely for the box to land on its right face or its left face because they are the same size.

   c. The box is more likely to land on either the front face or the back face than on any one of the other faces because these two are the largest of the faces.

d. Answers will vary. Sample data are given.

Outcomes	Tallies	Frequency
Front Face	𝍸 𝍸	10
Back Face	𝍸 |||	8
Top Face	|	1
Bottom Face	|	1
Right Face		0
Left Face		0
	Total	20

e. Based on the sample data, the prediction was true. The box landed on the front face and back face much more often than it landed on the other faces.

3. a. It is unlikely that the pencil will stop with the printed face on top because only one of its six faces has printing.

b. The pencil will stop with the printed face on top about five times because that is one-sixth of the 30 rolls $\left(\frac{1}{6} = \frac{\boxed{5}}{30}\right)$.

c. Answers will vary. Sample data are given.

Outcomes	Tallies	Frequency
Printed Face on Top	𝍸	5
Blank Face on Top	𝍸 𝍸 𝍸 𝍸 𝍸	25
	Total	30

d. Based on the sample data, the results were exactly as predicted! The pencil stopped with the printed face on top exactly five times.

4. A regular die looks more like a jelly powder box because they are both rectangular prisms and can land on any one of their six faces when tossed. However, the six outcomes of tossing a regular die and the six outcomes of rolling a hexagonal pencil are all equally likely, but the six outcomes of tossing a jelly box are not all equally likely.

Activity 3

1. **a.** About $\frac{2}{3}$ of the spinner is green and about $\frac{1}{3}$ of it is red.

   b. If I spun it 24 times, I would expect the pointer to stop on red about eight times because $\frac{8}{24} = \frac{1}{3}$.

2. **a.** The chances of the pointer stopping on the hearts or on the stripes are equally likely because each pattern covers $\frac{1}{2}$ of the spinner.

   b. If I spun it 50 times, I would you expect the pointer to land on the hearts about 25 times because $\frac{25}{50} = \frac{1}{2}$.

3. For each time the pointer stops on white, I would expect it to stop on blue three times because the blue area is about three times the white area. (About $\frac{1}{4}$ of the spinner is white and about $\frac{3}{4}$ of the spinner is blue.)

4. I agree with Dianne. The part covered by the stars is about $\frac{1}{6}$ of the area and the chance of tossing a 5 on a regular die is one out of six.

5. **a.** You are more likely to spin 30 minutes than 15 minutes. You are more likely to spin 30 minutes than 10 minutes. This is because the 30-minute parts cover more of the spinner than either one of the other two times.

   b. Your chances of spinning 30 minutes or of spinning a different time are equally likely. There are six equal sized regions on the dial and 3 regions are labelled "30 min." The other three regions are labelled other times.

c. Answers will vary. Sample data are given.

Outcomes	Tallies	Frequency
10 Minutes	𝍸	2
15 Minutes	𝍸 |	6
30 Minutes	𝍸 𝍸 ||	12
	Total	20

d. These answers are based on the sample data given in 5.c. Your results should be similar.

- My prediction in 5.a. was true. I spun 30 minutes more times than I spun 15 minutes and more times than I spun 10 minutes.
- My prediction in 5.b. was close to being true. Half of 20 spins is 10 spins, and I spun 30 minutes 12 times.

6. a. You are more likely to spin popcorn than muffins, and you are more likely to spin popcorn than cookies. This is because the popcorn parts cover more of the spinner than either one of the other two treats.

b. Your chances of spinning popcorn or spinning a different treat are equally likely because the popcorn parts cover half of the spinner.

c. Answers will vary. Sample data are given.

Outcomes	Tallies	Frequency
Popcorn	𝍸 𝍸 |	11
Muffins	𝍸 ||	7
Cookies	||	2
	Total	20

d. These answers are based on the sample data given in 6.c. Your results should be similar.

- My prediction in question 6.a. was true. I spun popcorn more times than I spun muffins and more times than I spun cookies.
- My prediction in question 6.b. was very close to being true. Half of 20 spins is 10 spins, and I spun popcorn 11 times. Yet, all of my results weren't what I expected. I thought I would spin muffins and cookies an equal number of times (about 4 or 5 each), but this didn't happen.

7. a. You are more likely to spin 30 minutes of homework because the total area covered by the 30-minute parts is greater than half the area of the spinner.

b. About $\frac{1}{8}$ of the spinner shows 60 minutes.

c. Going clockwise from the 60-minutes part, if you change the next two parts (30 minutes and 45 minutes) to 60 minutes, the pointer will likely stop on 60 minutes half of the time.

8. This is not fair because the green area is much more than twice the blue area.

9. The participant must twirl the spinner so that the pointer makes at least one complete revolution.

Challenge Activity

Spinner 2 will give Jim his best chance of getting quickly to the sack of gold. To move ahead 16 spaces using the least number of spins, Jim needs to make four spins of white (each spin moves 5 spaces forward) and two spins of blue (each spin moves 2 spaces back). He needs to spin white twice as many times as he spins blue, and on Spinner 2 the white area is twice as large as the blue area.

Lesson 3: Chance—Past and Present

Activity 1

1. **a.** It seems like it is a fair game because everything is based on chance.

   b. I can test my answer by playing the game over and over to see if both players win about the same number of times.

   Note: You may choose to repeatedly run the simulation of this Mayan game found at the following website:

   http://www.halfmoon.org/bul.html

2. Sample data are provided.

Colour	Who Won the Round? (tallies)	Total Rounds Won
Blue	𝍸 𝍸 ||	12
Red	𝍸 |||	8

3. This seems to be a fair game because they both won about ten rounds. I suppose I could play it several more times and see if they both win about the same number of times.

4. It is possible to complete a round in as few as two tosses.

5. I suppose it is possible to keep tossing forever and never get a winner.

6. The outcome of this game is not influenced by skill or experience. The two players have an equally likely chance of winning.

Activity 2

Sample answers are given.

1. I think A, E, I, O, and R are the five letters used most often in the English language.

2. I think X is the letter used least often in the English language.

3. **a.** There are about 1000 letters on page 187.

   b. I think this would be a suitable sample to use to check my predictions because the words appear to be everyday words, and there are lots of them.

   c.

	Frequency of Letters on page 187	Total
A	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	62
B	𝍸 𝍸 𝍸	15
C	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
D	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40
E	𝍸 𝍸	110
F	𝍸 𝍸 𝍸 |	16
G	𝍸 𝍸 𝍸 𝍸 𝍸	25
H	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	43
I	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	36
J		
K	𝍸 𝍸 𝍸	15
L	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	42
M	𝍸 𝍸 𝍸 𝍸	20
N	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	61

O	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	90
P	𝍸 𝍸 ||	12
Q		
R	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	49
S	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	59
T	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	67
U	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	47
V	𝍸	5
W	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	30
X		
Y	𝍸 𝍸 𝍸 𝍸 𝍸 ||||	29
Z	|	1
	Total	905

4. a. The letters on page 187, from greatest frequency to least frequency, are E, O, T, A, N, S, R, U, H, L, D, I, C, W, Y, G, M, F, B, K, P, V, and Z. The letters J, Q, and X were not in the sample.

b. I think my predictions were pretty good. For example, three of my predicted letters, A, E, and O, were the same as the sample. I also predicted I and R. R is the seventh letter in the sample and I is the twelfth letter in the sample.

5. a. I think the letter frequencies in a book I am reading or in a newspaper article would be similar to the textbook sample.

b. As my sample, I used page 29 in the book *Spirit in the Rainforest* by Eric Wilson.

	Frequency of Letters in Chosen Sample	Total
A	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	75
B	𝍸 𝍸 ||||	14
C	𝍸 𝍸 𝍸 𝍸 ||	22
D	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	48
E	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	104
F	𝍸 𝍸	10
G	𝍸 𝍸 𝍸 |||	18
H	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	57
I	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	63
J	|	1
K	𝍸 𝍸 𝍸 𝍸 ||	22
L	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	38
M	𝍸 𝍸 𝍸 |||	18
N	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	71
O	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	72
P	𝍸 𝍸 |||	13
Q	|	1

R	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	52
S	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	50
T	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	87
U	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	32
V	𝍸 𝍸 𝍸 ||	17
W	𝍸 𝍸 𝍸 ||||	19
X	|	1
Y	𝍸 𝍸 𝍸 𝍸	20
Z	|	1
	Total	926

6. a. The letters in my sample, from greatest to least frequency, are E, T, A, O, N, I, H, R, S, D, L, U, C, K, Y, W, G, M, V, B, P, F, J, Q, X, and Z.

b. The frequencies in my sample are similar to the textbook sample. The letter E was the most frequent letter in both samples. The next four most frequent letters in both samples were the same, although they were not in the same order. The letter Z appeared only once in each sample. The letters J, Q, and X were not in the textbook sample, and each of them appeared only once in my sample.

7. a. I would make the following letter tiles.

Letter	E	O	T	A	N	S	R	U	H	L	D	I	C
Number	12	8	7	7	6	6	6	5	5	5	5	5	3

W	Y	G	M	F	B	K	P	V	Z	J	Q	X
3	2	2	2	1	1	1	1	1	1	1	1	1

b. Scrabble® uses the following 98 tiles (plus 2 blank tiles).

Letter	E	A	I	O	N	R	T	D	L	S	U	G	B
Number	12	9	9	8	6	6	6	4	4	4	4	3	2

C	F	H	M	P	V	W	Y	J	K	Q	X	Z
2	2	2	2	2	2	2	2	1	1	1	1	1

My letter tiles are similar to the Scrabble® tiles. I had the same number of tiles as Scrabble® for 11 of the letters: E, O, R, D, Y, M, K, Z, J, Q, and X. Most of the others were pretty close. The two letters that differed the most were H and I. I had three more H's and four less I's than Scrabble®.

Activity 3

1. The population is all the milk chocolate M&M's® that are made.

2. Milk chocolate M&M's® are blue, brown, green, orange, red, and yellow.

3. A sample answer is given.

a. I think red will have the greatest frequency.
b. I think brown will have the least frequency.

4. A sample answer is given.

Colours of M&M's®	Number of Each Colour in My 48-g Sample (tallies)	Total Frequency
Blue	𝍸 ||	7
Brown	𝍸 𝍸 𝍸 𝍸 |	21
Green	||||	4
Orange	𝍸 ||	7
Red	|	1
Yellow	𝍸 𝍸 𝍸 ||||	19
	Total	59

Colours of M&M's®	Number of Each Colour in My 159-g Sample (tallies)	Total Frequency
Blue	𝍸 𝍸 𝍸 𝍸	20
Brown	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III	53
Green	𝍸 𝍸 𝍸 𝍸 I	21
Orange	𝍸 𝍸 𝍸 III	18
Red	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III	33
Yellow	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 II	32
	Total	177

5. A sample answer is given.

a. No, my prediction wasn't correct for either of my samples. The most frequent colour in both samples was brown.

b. No, my prediction wasn't correct for either of my samples. The least frequent colour in the 48-g sample was red. The least frequent colour in the 159-g sample was orange.

6.

Colours of M&M's®	Fraction of Each Colour	Number Expected in a 48-g Package (with 60 M&M's®)	Number Expected in a 159-g Package (with 180 M&M's®)
Blue	0.1	6	18
Brown	0.3	18	54
Green	0.1	6	18
Orange	0.1	6	18
Red	0.2	12	36
Yellow	0.2	12	36

7. Sample answers are given.

   a. In my 48-g sample, the numbers of blue, brown, green, and orange are close to the expected numbers. I only have one red compared to the expected number of 12 and 19 yellow compared to the expected number of 12.

   b. In my 159-g sample, the numbers of all the colours are close to the expected numbers. There are 18 orange, which is exactly the expected number.

8. A sample answer is given.

   My larger sample predicts the population better than my smaller sample. Most colours in both samples were close to expected numbers, but two colours in my 48-g sample were not close at all.

9. Every package of milk chocolate M&M's® in a particular size doesn't have the same number of each colour because the candies are mixed in very large amounts before the packages are filled.

10. Sample answers are given.

    I think the company changed to colours that they thought their customers would like better. If people like the looks of the candies, maybe they will buy more.

Challenge Activity

1. THE TROUBLE WITH USING A CODE LIKE THIS ONE IS THAT IT IS EASY FOR OTHER PEOPLE TO SOLVE IF THEY KNOW ABOUT LETTER FREQUENCIES.

2. This is the last challenge problem. I hope you have enjoyed solving them.

Keystrokes

1. If I am playing with a partner, this is a fair game because the numbers we can erase depend on the toss of the die, and we are equally likely to toss any number.

2. a.

Game	1	2	3	4	5	6	7	8	9	10
Number of Tosses Needed	10	21	12	13	19	12	15	21	17	12

b. In my results, the least number of tosses needed was ten.

c. In my results, the greatest number of tosses needed to win was 21.

d. The least number of tosses needed to win the game is six. The die has six different numbers and you would have to toss a different number each time. (It is quite unlikely for this to happen, because typically, it might take a person 14 tosses to finish.)

e. It is possible, but highly improbable, that you could toss forever and never get all six numbers.

f. The goal of ten for a solitaire game is reasonable because it is possible, but doesn't happen very often. In my results, it only happened once in ten games. It's fun to win, but if you win too often, it can be boring!

Review

1. **a.** When you see a halo around the moon, it is likely to rain very soon.
   b. If it is very cold in November, the rest of the winter will likely be mild.
   c. If the moon looks red, it will probably rain.

2. **Textbook, page 223, Skill Bank Looking Back, question 4**

   a. possible
   b. not likely
   c. likely
   d. certain
   e. impossible

3. **Textbook, page 196, Skill Bank from This Unit, question 2**

   It is possible that it will be windy tomorrow.

   Tuesday is always the day following Monday.

   I will never see a real live dinosaur.

   It is probable that I will read a story before I go to sleep tonight.

4. **a.** In Alberta, it is less likely to snow in June than in November.

   b. It is more probable that I will be awake at 3:00 **P.M.** than at 3:00 **A.M.** tomorrow.

   c. It is equally likely that the next person to enter a drug store will be male or female.

5. **a.** It is impossible that I will travel to the moon tomorrow.
   b. It is likely that I will have lunch tomorrow.
   c. It is improbable that I will find a penny today.
   d. It is certain that I will read a sentence today.

6. **a.** Milk will likely go sour if **it is left out of the fridge for a long time**.
   b. It is certain that tomorrow **will be 24 h long**.
   c. It is improbable that I will be outside **at midnight**.
   d. Cats never **speak French**.

7. **Textbook, page 196, Skill Bank from This Unit, question 3**

   Spinner A: Yellow and blue are equally likely.

   Green is more likely than red, yellow, or blue.

   Red is less likely than green, yellow, or blue.

   An even number is more likely than an odd number.

   Spinner B: All four colours are equally likely.

8. Textbook, page 186, Practise Your Skills, questions 1 to 4

1. Spinner B is more likely to spin red.
2. Spinner C is as likely to spin red as blue.
3. Spinner B is less likely to spin blue than red.
4. Spinner B and Spinner C will never spin yellow.

9. Textbook, page 196, Skill Bank from This Unit, question 1

Spinner A matches Tally chart 3. Yellow should have about half of the total tallies because half of the circle is yellow. Red should have about the same number of tallies as blue because they are both one-fourth of the circle. Spinner B matches Tally chart 2. Each of the three colours should have about an equal number of tallies because the circle is broken into three equal parts. Spinner C matches Tally chart 1. Yellow should have the fewest tallies because yellow is the smallest part. Red should have about the same number of tallies as blue because their parts are equal in size.

10. Textbook, page 243, Skill Bank Looking Back, question 1

Enzio used spinner B. He spun each colour about the same number of times and all sections on spinner B are the same size.

Sophie used spinner A. She spun white about as many times as her combined spins for red and blue, and the two white sections on spinner A are almost the same as the red and blue sections combined.

Abbas used spinner C. He spun blue the least number of times and white the greatest number of times, and on spinner C, the blue section covers the least area and the white sections cover more area than the blue or the red.

11. Textbook, page 186, On Your Own, question 1

Spinning blue and spinning yellow are equally likely.
Spinning red and spinning green are equally likely.
Spinning blue is more likely spinning red or spinning green.
Spinning yellow is more likely than spinning red or spinning green.
Spinning red is less likely spinning blue or spinning yellow.
Spinning green is less likely spinning blue or spinning yellow.

12. Textbook, page 223, Skill Bank Looking Back, question 3

Spinner A is as likely to spin red as blue.
Spinner B is less likely to spin red than blue.

13. Textbook, page 190, Skill Bank Looking Back, question 1

Spinner C was likely used to collect the data. Yellow and red are equally likely, and each is about twice as likely as blue.

14. a. I agree. It is improbable that the company makes the same number of each colour because the numbers of the different colours in the sample vary greatly.

b. I disagree. It's possible that the company makes more orange jelly beans than any other particular colour, but I cannot be certain because the sample had a lot of red, yellow and black, as well.

c. I agree. There weren't many pink, white or purple jelly beans, and there were about the same number of each of those colours.

15. Harley's sample most likely represents the population. The words on Van's dictionary page might all begin with the same letter, and it may be an infrequently used letter. Sooni's page may show the letter z more frequently than in most English writing because it is about zebras in the zoo.

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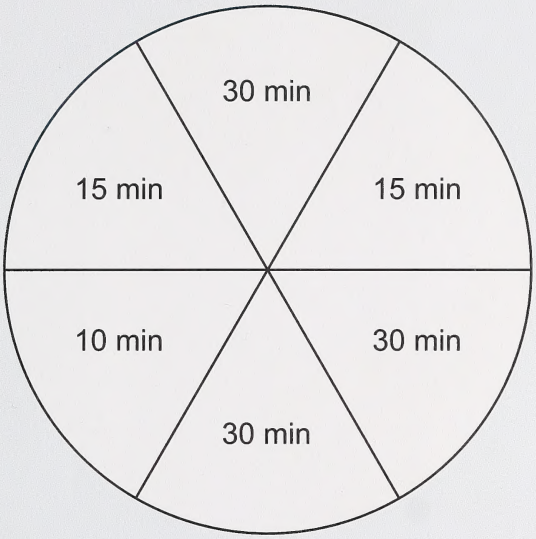
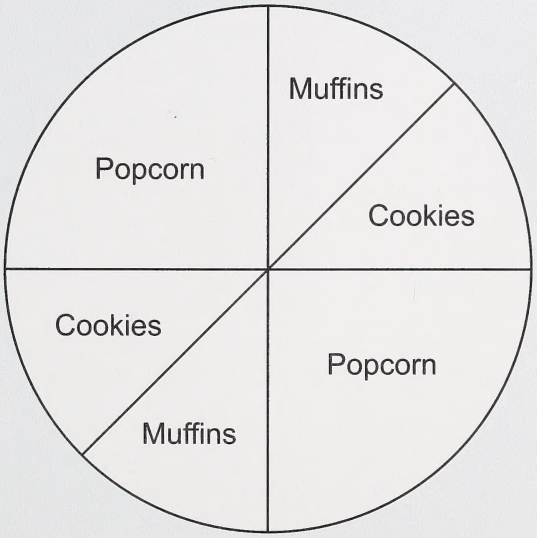
Cut-Out Learning Aids

Spinners





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